# Looking Inside XX Cyg

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**Abstract** Light curves of XX Cyg, an SX Phe type variable in the constellation Cygnus, are obtained using visible and blue filters. From the light curves, we obtain temperature, radius, and radial velocity throughout the pulsation period. Assuming the pulsation to be purely radial, the linear adiabatic wave equation can provide a limit on pressure scale height at the surface of the star. Our high school students are exposed to information about this star through the 'lens' of a telescope and through the 'lens' of an equation.

**Introduction**

XX Cyg (RA=20:03:15.6, Dec=+58:57:17.0, spectral type A5-F5) is an SX Phe star. It was discovered to be a variable by Ceraski (1904) and such stars have the following characteristics: short periods (<0.3 days), amplitudes (>0.1 mag), metal-poor Population II stars. They are generally found in the region where the instability strip intersects the main sequence in the H-R diagram and are slow rotators with one or two dominant radial modes.

**Photometry**

Photometric observations of XX Cyg were made with a 2 m Faulkes telescope located in Hawaii during July 2013. The telescope is equipped with a standard Johnson-Cousin-Bessel multicolor filter system and a Merope E2V 42-40 CCD camera. The CCD camera has 2K x 2K 13.5 micron pixels that correspond to a field of view of 4.7 arcmin. Figure 1 shows an image of XX Cyg taken with this telescope where two companion stars are marked. Photometry was performed using Aperture Photometry Tool (Laher et al. 2012a,b).


*Figure 1: CCD image taken with 2 m Faulkes telescope. North is up and east to the left. XX Cyg and companion stars are marked.*

The light curves were then produced by computing the magnitude difference between XX Cyg and one of the companion stars whose magnitude was obtained from AAVSO charts. Using this magnitude of XX Cyg we then computed the magnitude of the other standard stars and compared it to the AAVSO results to estimate our photometric precision. The standard deviations of the magnitude differences is $0^m.05$ .We compare our results with Yang et al. (2011) who took photometric observations of XX Cyg from 2007-2011. After a spectral analysis, Yang et al. (2011) found that their light curves are well fitted by the formula:

$$m = m_0 + \sum A_i \sin(2\pi f_i t)$$

The values of the amplitudes and frequencies are taken from their paper and correspond to a fundamental frequency and its 19 harmonics. The fundamental frequency is the reciprocal of the period which is 3.2368 hrs. We also compared our result with Berry et al.

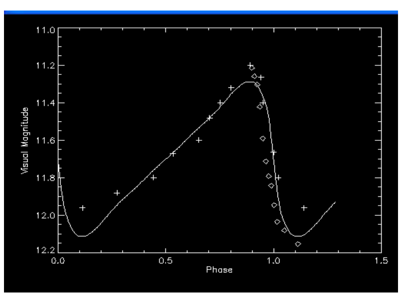


*Figure 2: Light curve of XX Cyg. Full line = results of Yang et al. (2011); diamonds= our results; crosses = results of Berry et al.*

The amplitudes varied slightly: 0.825 (Yang et al. 2011) , ours 0.94, and 0.76 (Berry et al.). Since the period of XX Cyg is known to change with time, one may expect the light curve to change. Some of the differences are beyond our standard deviation error and we plan to take follow-up observations.

The average B-V value reported (Berry et al.) is 0.3 and this corresponds to a temperature of 7385 K.

| Source | Parallax Angle | Absolute Magnitude | Radius of XX Cyg |
|---|---|---|---|
| SIMBAD | 2.08 | 3.13 | 1.293 $R_\odot$ |
| Yang et al. (2011) from photometry | | 1.31 | 2.99 $R_\odot$ |
| Hipparcos | 1.48 | 2.72 | 1.562 $R_\odot$ |

**Determination of Pressure Scale Height**

Near the surface of the star, and assuming that the ratio of specific heats is constant, the linear adiabatic adiabatic wave equation becomes (Cox 1980):

$$\frac{d^2\xi}{dr^2} - \frac{1}{H}\frac{d\xi}{dr} + \frac{\rho_0\omega^2}{\Gamma p_0}\xi = 0$$

In the above equation, ξ is the amplitude, *r* is the distance from the center of the star, *H* is the pressure scale height, $\rho_0$ and $p_0$ are the equilibrium density and pressure respectively, Γ the ratio of specific heats, and ω the angular frequency. For real wave numbers,

$$\frac{1}{H^2} > \frac{4\rho_0\omega^2}{\Gamma p_0}$$

If this inequality is combined with the ideal gas equation, and assuming a molecular weight of 0.0024 kg/mole, and the ratio of specific heats equal to the ideal gas value of 5/3, the limit on the pressure scale height is 5982 km (maximum).

**Determination of Polytropic Index *n* that best describes XX Cygni**

The Lane-Emden equation Chandrasekhar (1939) is given as

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\zeta}\right) = -\theta^n$$

In the above equation, $r = \alpha\zeta$ where α is a length constant and can be determined (see below) and $p = p_c\theta^{n+1}$ and $\rho = \rho_c\theta^n$ ; the subscripts *c* refer to the center of the star. We solved for different values of polytrope index *n* and checked our results with Rodriguez & Breger (2001). The pressure scale height is then given by:

$$\frac{1}{H} = -\frac{1}{p}\frac{dp}{dr} = -\frac{n+1}{\alpha}\frac{1}{\theta}\frac{d\theta}{d\zeta}$$

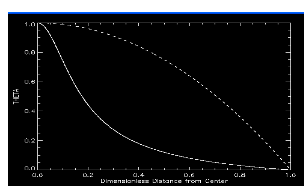


*Figure 3: Plot of θ versus dimensionless distance from center. The broken curve is for n=0 and the full line for n=4. The gradient for n=4 is greater at the center of the star compared to n=0 and lower than n=0 at the surface.*

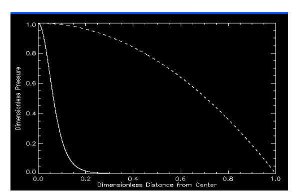


*Figure 4: Dimensionless pressure versus dimensionless distance from the center. The dotted line is for n=0 and the full line is for n=4. As the polytropic index increases, the gradient of pressure at the center becomes steeper.*

We used our solution to obtain the pressure scale height for various values of n and our results are :

| n | ζ | r/R | θ | $\frac{d\theta}{d\zeta}$ | H (km) |
|---|---|---|---|---|---|
| 0 | √6 | 0.95 | 0.0975 | -0.775 | 46 220 |
| 1 | 3.142 | 0.95 | 0.0524 | -0.3485 | 21 535 |
| 2 | 4.35 | 0.95 | 0.0295 | -0.141 | 14 408 |
| 3 | 6.897 | 0.95 | 0.0154 | -0.047 | 10 688 |
| 3.5 | 9.53581 | 0.95 | 0.0104 | -0.02304 | 9 499 |
| 4 | 14.972 | 0.95 | 0.00631 | -0.00888 | 8 538 |

The values in the second column are taken from Rodriguez & Breger (2001). From these results it is clear that our star is best described by a polytrope of index of 4 or greater.

**Determination of Mass of XX Cygni**

The linear adiabatic wave equation (LAWE) has been solved for various polytropic models (Berry et al.) and the mass of XX Cygni can be obtained from these solutions.

| Polytrope model | SIMBAD | Yang et al. (2011) | Hipparcos |
|---|---|---|---|
| *n*=0 | 1.63 $M_\odot$ | 20.2 $M_\odot$ | 2.9 $M_\odot$ |
| *n*=1 | 0.86 $M_\odot$ | 10.7 $M_\odot$ | 1.5 $M_\odot$ |
| *n*=1.5 | 0.60 $M_\odot$ | 7.4 $M_\odot$ | 1.1 $M_\odot$ |
| *n*=2 | 0.41 $M_\odot$ | 5.0 $M_\odot$ | 0.7 $M_\odot$ |
| *n*=3 | 0.18 $M_\odot$ | 2.2 $M_\odot$ | 0.31 $M_\odot$ |
| *n*=3.5 | 0.13 $M_\odot$ | 1.6 $M_\odot$ | 0.23 $M_\odot$ |
| *n*=4 | 0.11 $M_\odot$ | 1.3 $M_\odot$ | 0.19 $M_\odot$ |

Since the masses of the SX Phe stars are known to be in the range of 1.5 -2.5 $M_\odot$, it appears that Yang et al. (2011) and model of *n* around 3.5 and greater can be used to describe XX Cygni. Future work will be directed to solving LAWE for models between 3.5 and 4.5.

**Conclusions**

We have used LAWE and telescopic observations to determine the pressure scale height of XX Cygni as 5982 km. Polytrope models with *n* =3.5 or greater when used in conjunction with LAWE give the mass of XX Cygni to be in the range of 1.3 -1.6 $M_\odot$.

**References**

Berry et al., http://www.wvi.com/~rberry/astronomy/xxcygni/xxcygni.htm
Ceraski, W. 1904, Astron. Nachr., 165, 61
Chandrasekhar, S. An Introduction to the Study of Stellar Structures (Chicago: University of Chicago Press, 1939)
Cox, J.P., Theory of Stellar Pulsation (Princeton, New Jersey, 1980).
Laher, R., et al., 2012a, PASP, 124, 737
Laher, R., et al., 2012b, PASP, 124, 764
Rodriguez, E., Breger, M. 2001, A& A. 306, 178
Yang, X.H., Fu, J.N., Zha, Q. 2011, A.J. 144, 92